Kinetics of histaminase

SIR,—Spencer (1963) outlined a method for the determination of relative concentrations of histaminase. In his experimental work he found that the plot of $\log_{10} (x_0/x_t)$ against t is not strictly linear (x_0 initial concentration of substrate, x_t concentration at time t). If the reaction involved is of the form:

$$\mathbf{S} + \mathbf{En} \stackrel{\mathbf{k_1}}{\approx} \mathbf{S} \cdot \mathbf{En} \stackrel{\mathbf{k_3}}{\approx} \mathbf{P} \cdot \mathbf{En} \stackrel{\mathbf{k_5}}{\approx} \mathbf{En} + \mathbf{P}$$

where S denotes substrate molecule, P product molecule and En enzyme molecule, it can be shown that $-1/e.d[S]/dt = (k_a[S]-k_b[P])/(k_c[S] + k_d + k_e[P])$ where k_a , k_b , k_c , k_d , and k_e are each composites of some of the velocity constants above, square brackets denote molar concentrations and e is the concentration of enzyme. But -1/e.d[S]/dt = 1/e.d[P]/dt and $[S] = [S_0]$ when t = 0, so $[P] = [S_0] - [S]$ and the rate equation takes the form $-1/e.d[S]/dt = (k[S]-k^1[S_0])/([S] + K))$ where $k + k^1$ are constants, and for a particular value of $[S_0]$, K also is constant. The reaction may be effectively irreversible if any of the backward velocity constants are sufficiently small since k^1 is directly proportional to $k_2.k_4.k_6$ and hence the rate equation becomes -1/e.d[S]/dt = k[S]/([S] + K)) which is formally the same as the Michaelis equation (1913) though the constants have different meanings. Integration and evaluation of the integration constant by insertion of $[S_0]$ for [S] when t = 0 gives the equation

which differs from that used in the original paper solely in the presence of the term $([S_0]-[S])$ and thus accounts for the lack of linearity. The dependance of K on $[S_0]$ except when $k_6 = 0$ may account partially for dependance of the results on $[S_0]$ particularly after long times of incubation.

If t_{α} is the time at which $\alpha \%$ of the substrate has been consumed

k e t
$$-\alpha[S_0] = 2.303$$
 K $\log_{10}(1/1-\alpha) = a$ constant for a given value of α (2)

Thus if $t_{\alpha 1}$, and $t_{\alpha 2}$ are the corresponding times in incubation mixtures containing enzyme concentrations of e_1 and e_2 respectively but the same initial concentration [S₀] of substrate

$$k e_1 t_{\alpha 1} - \alpha [S_0] = k e_2 t_{\alpha 2} - \alpha [S_0] \text{ or } e_1/e_2 = t_{\alpha 2}/t_{\alpha 1}. \qquad (3)$$

and hence the method of determination in the original paper is theoretically sound despite the non-linearity of the plot used which is purely empirical.

The data of the paper provides a further test of the equations: if two different percentages of consumption are used the direct effect of variations of $[S_0]$ can be eliminated. From (2) for incubation mixtures containing initially $[S_1^0]$ of substrate, e_1 of enzyme; and $[S_2^0]$ of substrate and e_2 of enzyme.

For percentage
$$\alpha$$
 consumption k e₁ t $_{\alpha 1} - \alpha[S_1^0] = k e_2 t \alpha_2 - \alpha[S_2^0] \dots$ (4)

For percentage β consumption k e₁ t $_{\beta_1} - \beta[S_1^0] = k e_2 t _{\beta_2} - \beta[S_2^0]$. (5)

whence $\beta \times (4) - \alpha \times (5)$ gives

Table 1 gives the application of this equation to the data of Fig. 4 of the original paper.

[S ₀], (μg/ml)			1.53	2.01	2.49
$\alpha = 50, t_{\alpha}$ (min.)	••		25	26	27
$\beta = 25, t_{\beta} \text{ (min.)}$			10	10.5	11
$(\beta t_{\alpha} - \alpha t_{\beta})$			125	125	125
Ratio of enzyme con	centrat	ions			
(calc.)			1.00	1.00	1.00
]		

Equation (6) may be used also for incubation mixtures containing the same $[S_0]$. Table 2 shows its application to the data in Fig. 3 of the original paper.

TABLE 2

Ratio of enzyme	concentrations		ions				
(pre-arranged)	••			1.30	1.00	0.80	0.20
$\alpha = 50, t_{\alpha} \text{ (min.)}$				22	28	34	53
$\beta = 25, t_{\beta} \text{ (min.)}$		••		9	11.5	14	22
$(\beta t_{\alpha} - \alpha t_{\beta})$	••			100	125	150	225
Ratio of enzyme concentrations (calc.)			1.25	1.00	0.83	0.55	
-					I		

In practice using equation (6) to determine relative concentrations of enzyme would require greater experimental accuracy than keeping $[S_0]$ constant and using equation (3). There is a trend of deviation in the results of the calculations in Table 2 and in those of Table 1 of the original paper which may be due to a dependence of K on $[S_0]$ so that equation (2) is not strictly equal to a constant: the effect is rather greater when (6) is used. This can be allowed for by replacing K by $(K - \theta[S_0])$, where θ is a constant, in equation (2) with the result that equation (3) becomes $(e_1 + \psi[S_0])/(e_2 + \psi[S_0]) = t_{\alpha 2}/t_{\alpha 1}$, where ψ is a constant. Substituting the data of Table 1 of the original paper for incubation mixtures A and B ($e_2 = 1.30$ inits, $e_1 = 1.00$ units, t $_{\alpha 2} = 22$, t $_{\alpha 1} = 28$, $\alpha = 50$) gives $\psi[S_0] = 0.10$. Thus, for another incubation mixture using the same $[S_0]$, e may be calculated in the same units from $(e_1 + 0.10)/(e + 0.10) = t_{\alpha}/t_{\alpha 1}$ or, in this case $e = (30.8/t_{\alpha}) - 0.10$. This gives e = 0.80 and 0.48 respectively for incubation mixtures C and D of the original paper: they were set up to be 0.80 and 0.50respectively and the uncorrected calculation gives 0.82 and 0.53 respectively. Thus the accuracy of the results can be improved by using two control incubation mixtures in which the concentrations of the enzyme are in a known ratio.

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References

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